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# Application of thermal resistance network model in optimization design of micro-channel cooling heat sink

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## Abstract

**Purpose** – The purpose of this paper is to optimize the configuration sizes of micro-channel cooling heat sink using the thermal resistance network model. The optimized micro-channel heat sink is simulated by computational fluid dynamics method, and the total thermal resistance is calculated to compare with that of thermal resistance network model.

**Design/methodology/approach** – Taking the thermal resistance and the pressure drop as goal functions, a multi-objective optimization model was proposed for the micro-channel cooling heat sink based on the thermal resistance net work model. The Sequential Quadratic Programming procedure was used to do the optimization design of the structure size of the micro-channel. The optimized micro-channel heat sink was numerically simulated by computational fluid dynamics (CFD) software.

**Findings** – For the heat sink to cool a chip with the sizes of $L \times W = 2.5\,\text{mm} \times 2.5\,\text{mm}$ and the power of 8 W, the optimized width and height of the micro-channel are 154 μm and 1,000 μm, respectively, and its corresponding total thermal resistance is 8.255 K/W. According to the simulation results, the total thermal resistance of whole micro-channel heat sink $R_{total}$ is 7.596 K/W, which agrees well with the analysis result of thermal resistance network model.

**Research limitations/implications** – The convection heat transfer coefficient is calculated approximatively here for convenience, and that may induce some errors.

**Originality/value** –The maximum difference in temperature of the optimized micro-channel cooling heat sink is 59.064 K, which may satisfy the requirement for removal of high heat flux in new-generation chips.

**Keywords** Thermal resistance, Fluid dynamics, Optimum design, Programming

**Paper type** Research paper

## 1. Introduction

With the developments of international aerospace technology, micro-electromechanical system and micro-machining technology, to transfer the heat generated by the micro-electronic chips to keep the stable and reliable operation of the devices is present problem to be solved in micro-electronic industry (Zeng-Yuan, 1988). When the heat flux of micro-electronic devices exceeds 100 W/cm$^2$, traditional cooling method is unlikely to meet the cooling needs (Kleiner *et al.*, 1995), and now micro-channel cooling

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heat sink is an effective kind of substitute method. Tuckerman and Pease (1981a, b) first proposed that micro-channel heat sink can be used to cool electronic devices, and studied cooling performance of micro-channels directly fabricated into the back of silicon wafers of electronic chip. They found that the frictional resistance coefficient of liquid in the micro-channels was higher than that predicted by classical theory. The primary reason that affects the heat transfer performance in micro-channel cooling heat sink is the total thermal resistance of heat sink, and thermal resistance network model is an effective analysis method for thermal resistance. Wei and Joshi (2003) developed a simple thermal resistance network model to evaluate the overall thermal performance of a stacked micro-channel heat sink, and the aspect ratio, fin thickness and the ratio of channel width to fin thickness was optimized based on the model. Skandakumaran *et al.* (2004) analyzed the thermal resistance of single and multi-layer micro-channel heat sinks with the thermal resistance network model. Chong *et al.* (2002) modeled a single layer counter flow and a double layer counter flow micro-channel heat sink with rectangular channels by employing the thermal resistance network., and the accuracy of the prediction was verified by comparing the results obtained with those from the more comprehensive three dimensional CFD conjugate heat transfer model, and good agreements were obtained. The results showed that the overall thermal resistance was related with configuration sizes of micro-channel heat sink. Shao *et al.* (2007) optimized the cross-section sizes of micro-channels, the heat flux of chip is $278\,\mathrm{W/cm^2}$, and through the optimization micro-channel cooling heat sink, the highest temperature in the chip could be kept below 42 °C. Quadir *et al.* (2001) applied a finite-element method to evaluate the performance of micro-channel heat exchangers, and the methodology was able to predict the surface temperature, the fluid temperature and the total thermal resistance of the micro-channel heat sink. Liu and Garimella (2005) provided modeling approaches of increasing levels of complexity for the analysis of convective heat transfer in micro-channels which offer adequate descriptions of the thermal performance. Hegde *et al.* (2005) analyzed two-phase flow in micro-channel heat exchangers by using the finite-element method to solve the energy balance equations developed for two-phase flow in micro-channels. Jeevan *et al.* (2005) used the genetic algorithms under different flow constraints to determine the optimal dimensions for a stacked micro-channel, and the 2D FEM analysis resulted in lower thermal resistance. Massarotti *et al.* (2003) investigated microscopic and macroscopic approaches to the solution of natural convection in enclosures filled with fluid saturated porous media, and at the microscopic level, the porous medium was represented by different assemblies of cylinders and the Navier-Stokes equations were assumed to govern the flow.

The heat transfer performance of micro-channel cooling heat sink is affected by the flow state of liquid in micro-channel, besides the thermal resistance. Different thermophysical properties and velocity for different liquid result different flow state, and different flow state results different heat transfer effect. Flow of liquid can be expressed as pressure drop, and pressure drop is related with configuration sizes of micro-channel heat sink. Therefore, based the effect of configuration of micro-channel heat sink on the thermal resistance and pressure drop, which can be as goal function, the problem becomes multi-objective optimization. The sequential quadratic programming (SQP) procedure was used to do the optimization design of the configuration size of the micro-channel. The optimized micro-channel heat sink is simulated by CFD method, and the total thermal resistance is calculated to compare with that of thermal resistance network model.

## 2. Physical model and computational zone

A micro-channel cooling heat sink is shown in Figure 1, which is used to cool a chip with the size of $W \times L = 2.5\,\text{mm} \times 2.5\,\text{mm}$, and the power is 8 W, so the corresponding heat flux is $128\,\text{W/cm}^2$. The working fluid is deionized water. In the Figure 1, $W_c$ is the width of micro-channel, $H_c$ is the depth of micro-channel, $W_w$ is the width of fin, and $H_{sub}$ is the thickness of substrate. For the symmetry of the structure of model and load, the computational zone can be half of micro-channel and fin, and the schematic diagram of computational zone cross section is shown in Figure 2.

## 3. Thermal resistance network model

In micro-channel cooling heat sink, the total resistance $R'_{total}$ includes the substrate conduction thermal resistance $R_{sub}$, the wall conduction thermal resistance $R_{wall}$, the substrate convection thermal resistance $R_{sub,conv}$, the wall convection thermal resistance $R_{wall,conv}$ and the liquid flow thermal resistance $R_{fluid}$. For the computational zone in Figure 2, the original thermal resistance network model and the equivalent thermal resistance network model are shown in Figure 3.

Thus, the total thermal resistance $R'_{total}$ of half micro-channel and fin can be expressed as:

$$R'_{total} = R_{sub} + \left( \frac{1}{R_{sub,conv}} + \frac{1}{R_{wall} + R_{wall,conv}} \right)^{-1} + R_{fluid} \qquad (1)$$

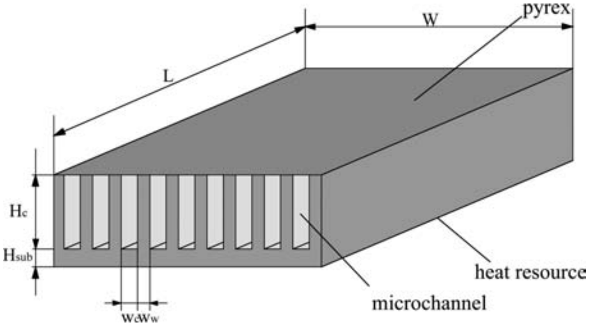


**Figure 1.**
Schematic diagram of three-dimension rectangle micro-channels cooling heat sink

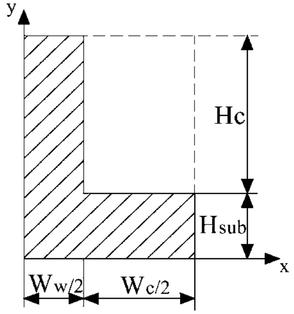
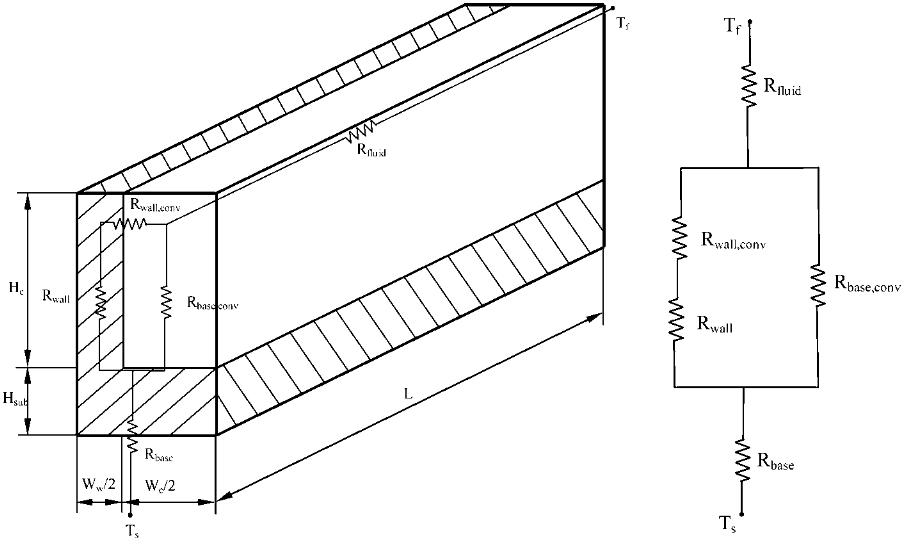


**Figure 2.**
Schematic diagram of computational zone cross section



(a) (b)

**Figure 3.**
Thermal resistance network model for half micro-channel and fin:
(a) original thermal resistance network model,
(b) equivalent thermal resistance network model

Where the substrate conduction thermal resistance $R_{sub}$ is

$$R_{sub} = \frac{H_{sub}}{k_s((W_c/2) + (W_w/2))L} \tag{2}$$

The wall conduction thermal resistance $R_{wall}$ is

$$R_{wall} = \frac{H_c}{k_s(W_w/2)L} \tag{3}$$

The wall convection thermal resistance $R_{wall,conv}$ is

$$R_{wall,conv} = \frac{1}{h_{conv}H_cL} \tag{4}$$

The substrate convection thermal resistance $R_{sub,conv}$ is

$$R_{sub,conv} = \frac{1}{h_{conv}(W_c/2)L} \tag{5}$$

The liquid flow thermal resistance $R_{fluid}$ is

$$R_{fluid} = \frac{1}{\dot{m}C_p} \tag{6}$$

In expressions (2)-(6), $K_s$ is the thermal conductivity of the substrate, $\dot{m}$ is the mass flow rate of working fluid in half of micro-channel, $C_p$ is the specific heat of the working fluid and $h_{conv}$ is the convective heat transfer coefficient.

$$h_{conv} = \frac{Nu \cdot k_l}{D_h}$$

where $K_l$ is the thermal conductivity of working fluid, $D_n$ is hydraulic diameter of cross section, for half of rectangle micro-channel, $D_h = 4A/P = W_c H_c/\ (W_c/2 + H_c)$, where $A$ is cross-sectional area of half of micro-channel, $P$ is wetted perimeter of half of micro-channel. When Re is less than 1,000, Nu can be express with experimental formula as (Kim and Kim, 1991)

$$Nu = 2.253 + 8.164\left(\frac{\alpha}{\alpha + 1}\right)^{1.5}$$

where $\alpha$ is ratio of height to width of micro-channel, $\alpha = H_c/W_c$.

Re can be calculated with the formula (7)

$$\mathrm{Re} = \frac{\rho_l u D_h}{\nu} \tag{7}$$

where $\nu$ is dynamic viscosity, $\rho_l$ is density of liquid, u is velocity of liquid. Considered the effect of temperature on dynamic viscosity, the formula can be used to calculate the dynamic viscosity of liquid (Incorpera, 1999).

$$\nu = 2.414 \times 10^{-5} \cdot 10^{247.8/(T-140)} \tag{8}$$

According to which (1), the total thermal resistance of the whole micro-channel heat sink can be expressed as

$$R_{total} = \frac{R'_{total}}{2n} \tag{9}$$

Where $n$ is the number of micro-channels.

Besides thermal resistance, pressure drop affects the heat transfer performance of micro-channel heat sink (Wu and Cheng, 2003).

$$\Delta P = 2f\rho_l u_{ave}^2 \frac{L}{D_h} \tag{10}$$

Where $u_{ave}$ is the mean velocity of liquid, $f$ is friction coefficient. For convenience of calculation (Li *et al.*, 2004), $f\mathrm{Re} \approx 68$, when $\mathrm{Re} < 200$.

## 4. Multi-objective optimization method

Multi-objective optimization problem deals with optimization problem to solve objective vector F(x), generally with constraints, the standard form is

$$\min_{\mathbf{x}\in R^n} \mathbf{F}(\mathbf{x}) = \min_{\mathbf{x}\in R^n}(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))$$

where

$$\mathbf{G}_i(\mathbf{x}) = 0 \quad i = 1, \ldots, m_e$$

$$\mathbf{G}_i(\mathbf{x}) \leq 0 \quad i = m_e + 1, \ldots, m$$

$$lb \leq \mathbf{x} \leq ub$$

The base idea to solve multi-objective optimization problem is to structure a merit function from each subgoal function, and translate multi-objective optimization problem into single-objective optimization problem to solve merit function with goal attainment programming. The problem formulation allows the objectives to be under- or overachieved, enabling the designer to be relatively imprecise about initial design goals. The relative degree of under- or overachievement of the goals is controlled by a vector of weighting coefficients, $\mathbf{w} = (w_1, w_2, \ldots, w_m)$, and is expressed as a standard optimization problem using the following formulation.

$$\min_{\gamma \in R, \mathbf{x} \in R^n} \quad \gamma \tag{11}$$

such that $F_i(\mathbf{x}) - w_i\gamma \leq f_i^*, i = 1, 2, \ldots, m$

In goal attainment programming there might be a more appropriate merit function, which you can achieve as the minimax problem.

$$\min_{\mathbf{x} \in R^n} \max_i \{\Lambda_i\} \tag{12}$$

where

$$\Lambda_i = \frac{F_i(\mathbf{x}) - f_i^*}{w_i}, \quad i = 1, 2, \ldots, m$$

Different optimization algorithm has different merit function. In SQP, the merit function Equation (12) can be written as

$$\psi(\mathbf{x}, \gamma) = \gamma + \sum_{i=1}^{m} r_i \cdot \max\{0, F_i(\mathbf{x}) - w_i\gamma - f_i^*\} \tag{13}$$

When the merit function of Equation (13) is used as the basis of a line search procedure, then, although $\psi(\mathbf{x}, \gamma)$ might decrease for a step in a given search direction, the function $\max \Lambda_i$ might paradoxically increase. A solution is therefore to set $\psi(\mathbf{x})$ equal to be

$$\psi(\mathbf{x}) = \sum_{i=1}^{m} \begin{cases} r_i \cdot \max\{0, F_i(\mathbf{x}) - w_i\gamma - f_i^*\} & \textit{if} \quad w_i = 0 \\ \max\limits_i \Lambda_i, i = 1, 2, \cdots, m & \textit{otherwise} \end{cases} \tag{14}$$

Suitable goal function and weighting coefficient are selected, multi-objective optimization problem can be optimized by using merit function Equation (14).

## 5. Optimization design

Select the number of micro-channel $n$, the width of micro-channel $W_c$, the width of fin $W_w$ and the height of micro-channel $H_c$ as design variable, expressed as $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ and

$\mathbf{x}_4$ respectively, and written in vector $\mathbf{x} = [x_1, x_2, x_3, x_4]$. The formula (1) and (10) are goal functions, and can be written in function of $\mathbf{x}$, as $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$. The width of heat sink are constraint, and the boundary of variables are construct, the multi-objective optimization problem is

$$\begin{cases} \textit{Find} & \mathbf{x} \\ \min & F(\mathbf{x}) = \min\{f_1(\mathbf{x}), f_2(\mathbf{x})\} \\ s.t. & x_1x_2 + (x_1+1)x_3 - 0.0025 = 0 \\ & 5 \le x_1 \le 300 \\ & 5 \times 10^{-5} \le x_2 \le 10^{-3} \\ & 2 \times 10^{-5} \le x_3 \le 10^{-3} \\ & 5 \times 10^{-5} \le x_4 \le 10^{-3} \end{cases} \tag{15}$$

The goal attainment SQP method is used to optimize the above problem. At each major iteration, an approximation is made of the Hessian of the Lagrangian function using a quasi-Newton updating method. This is then used to generate a QP subproblem whose solution is used to form a search direction for a line search procedure.

## 6. Optimization results and discussion

Initial value does not affect optimization results significantly, but suitable initial value can decrease iteration time. The initial value is chosen as $\mathbf{x}_0 = [10, 1 \times 10^{-4}, 8 \times 10^{-5}, 5 \times 10^{-4}]$ in this paper, the objective value of thermal resistance and pressure drop is $F^* = [9, 1]$, the units are K/W and 100 Pa, respectively. The weighting coefficient is chosen according to the relative importance between thermal resistance and pressure drop. Figure 4 shows the effect of weighting coefficient on thermal resistance and pressure drop (see Figure 5), where the thermal resistance is the total thermal resistance of the whole micro-channel heat sink. The thermal resistance increases with weighting coefficient, and pressure drop decreases with weighting coefficient, which indicates that the thermal resistance is less when the thermal resistance is more important than pressure drop. When the weighting coefficient is

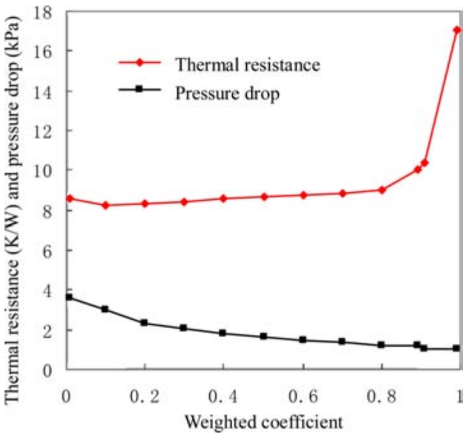


**Figure 4.** Effect of weighting coefficient on thermal resistance and pressure drop

about 0.1, the thermal resistance is least, and corresponding pressure drop is about 280 Pa, and corresponding configuration sizes of micro-channel are listed in Table I.

The total and component thermal resistance is listed in Table II, which show that the substrate conduction thermal resistance is minimal in total thermal resistance. The main reason is that micro-channels are fabricated directly into the back of chip, which eliminates the thermal contact resistance between micro-channel heat sink and chip. The liquid flow thermal resistance is maximum in total thermal resistance, which because the velocity of liquid is very low for the restriction of pump power. The total thermal resistance and pressure drop for whole micro-channel heat sink are listed in Table III.

## 7. Numerical simulation

The numerical simulation is used to verify the cooling performance of optimal micro-channel cooling heat sink by using electronics cooling software that has been previously used in analyzing the heat transfer character in electronics and chip cooling applications, which uses finite-volume method to solve CFD problem. For simplicity of

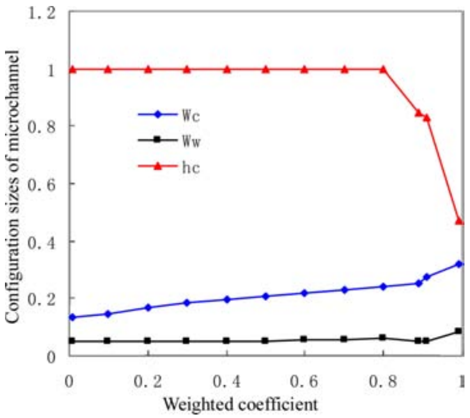


**Figure 5.**
Micro-channel sizes for different weighting coefficient

**Table I.**
Micro-channel sizes after optimizing

| $n$ | $W_c$ (μm) | $W_w$ (μm) | $W_c$ (μm) |
|---|---|---|---|
| 12 | 154 | 50 | 1,000 |

**Table II.**
Total thermal resistance and component thermal resistance for half of micro-channel and fin

| $R'_{total}$ (K/W) | $R_{base}$ (K/W) | $R_{wall}$ (K/W) | $R_{base,conv}$ (K/W) | $R_{wall,conv}$ (K/W) | $R_{fluid}$ (K/W) |
|---|---|---|---|---|---|
| 198.120 | 2.150 | 98.108 | 132.098 | 9.557 | 136.654 |

**Table III.**
Total thermal resistance and pressure drop for whole micro-channel heat sink

| $R_{total}$ (K/W) | ΔP (Pa) |
|---|---|
| 8.255 | 284.044 |

computational zone, hexahedron structured grids are used to mesh the computational zone. First order upwind scheme is used to discrete control equations, and semi-implicit method for pressure-linked equations is used to solve discretization equations.

Restricted by the power of pump, the inlet velocity of working fluid is 1 m/s, the density of working fluid is $\rho_f = 997\,\text{kg/m}^3$, the specific heat is $C_p = 4{,}179\,\text{J/kgK}$, the dynamic viscosity is calculated by Equation (8), the height between the chip surface and the base of the micro-channel $h_{sub}$ is 100 µm, the dimension of chip is $L \times W = 2.5\,\text{mm} \times 2.5\,\text{mm}$, and the thermal conductivity of the substrate $K_s$ and working fluid $K_f$ are 148 and 0.613 W/mK. The Reynolds number estimated by the above conditions is less than 200, therefore the flow is laminar.

Figure 6 shows the temperature distribution of optimized micro-channel for half channel and fin. The maximum difference in temperature is 59.064 K, and the transferred power of heat flux is 0.324 W, so the total thermal resistance $R'_{total}$ is 182.296 K/W, and the total thermal resistance of whole micro-channel heat sink $R_{total}$ is 7.596 K/W, which agree well with the analysis result (8.255 K/W) of thermal resistance

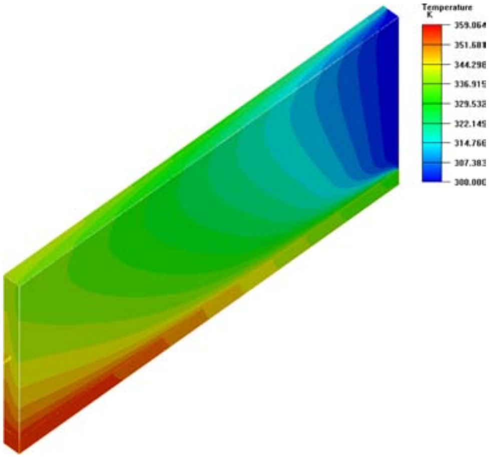


**Figure 6.**
Temperature distribution of optimized micro-channel

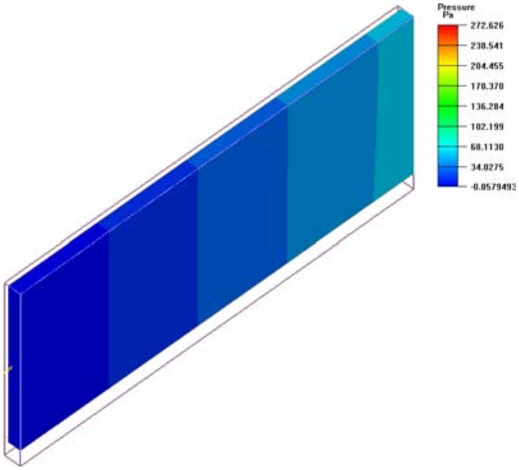


**Figure 7.**
Pressure distribution of optimized micro-channel

network model, and the relative error is 7.983 per cent. Figure 7 shows the pressure distribution of optimized micro-channel for half channel and fin. The pressure drop of liquid in micro-channel is 272.626 Pa, which agree well with the analysis result (284.044 Pa), and the relative error is 4.020 per cent.

## 8. Conclusion

Thermal resistance network model is used to analyze the thermal resistance of micro-channel heat sink, and establish thermal resistance network model for half micro-channel and fin. Based on the model and pressure drop formula, the multi-objective optimization model of micro-channel heat sink is found, and SQP is used to optimize the configuration sizes of micro-channel. The height and width of optimized micro-channel are 1,000 and 154 μm, and the ratio is 6.49, which is less than 10 (Ryu *et al.*, 2002). The thermal resistance of whole micro-channel heat sink is 8.255 K/W. To verify the optimization results, CFD numerical simulation method is used, and the results agree well with analysis results.

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